Frame-Independence of Exclusive Amplitudes in the Light-Front Quantization

Chueng-Ryong Ji and Chad Mitchell

Department of Physics, North Carolina State University, Raleigh, NC 27695-8202, USA

Abstract

While the particle-number-conserving convolution formalism established in the Drell-Yan-West reference frame is frequently used to compute exclusive amplitudes in the light-front quantization, this formalism is limited to only those frames where the light-front helicities are not changed and the good (plus) component of the current remains unmixed. For an explicit demonstration of such criteria, we present the relations between the current matrix elements in the two typical reference frames used for calculations of the exclusive amplitudes, *i.e.* the Drell-Yan-West and Breit frames and investigate both pseudoscalar and vector electromagnetic currents in detail. We find that the light-front helicities are unchanged and the good component of the current does not mix with the other components of the current under the transformation between these two frames. Thus, the pseudoscalar and vector form factors obtained by the diagonal convolution formalism in both frames must indeed be identical. However, such coincidence between the Drell-Yan-West and Breit frames does not hold in general. We give an explicit example in which the light-front helicities are changed and the plus component of the current is mixed with other components under the change of reference frame. In such a case, the relationship between the frames should be carefully analyzed before the established convolution formalism in the Drell-Yan-West frame is used.

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I. INTRODUCTION

The hadron phenomenology based on the equal-τ quantization takes great advantage of the Drell-Yan-West frame [1]. In the $q^+ = 0$ Drell-Yan-West frame [1], one can derive a first-principle formulation for the exclusive amplitudes by judiciously choosing the good component of the light-front current. Due to the rational dispersion relation in the equal-τ formulation, one doesn't need to suffer from the complicated vacuum fluctuation. The zero-mode contribution may also be avoided in the Drell-Yan-West frame by using the plus component of the current [2]. However, caution is needed in applying the established Drell-Yan-West formalism to other frames because in general the current components do mix under transformation of the reference-frame. Furthermore, the Poincare algebra in the ordinary equal-t quantization is drastically changed in the light-front equal-τ quantization. The upshot of the difference may be summarized as the change of the dynamical operators in the two quantizations. For example, the transverse rotation operator in the light-front quantization does not commute with the Hamiltonian possessing the dynamics. Likewise, the light-front helicities are not in general independent from the reference frame. Therefore, the adopted light-front formulation used in one particular reference-frame to compute physical quantities is not gauranteed to work if another reference-frame is used. Consequently, it is important to carefully check if the same formulation can indeed be used in another frame for the correct calculation insuring the frame-independence of the physical quantities.

In this work, we present criteria which insure an identical formulation in different frames of the light-front quantization. As an explicit example, we consider the well-known convolution formalism for the exclusive processes established in the Drell-Yan-West($q^+ = 0$) frame [1] and investigate the criteria to insure the same formalism in different reference-frames. In particular, we analyze the transformation of the electromagnetic current elements and the light-front helicities between the Drell-Yan-West frame and the Breit($q^+ = 0$) frame [3] which is another popular reference-frame used frequently by Frederico and collaborators for the calculation of exclusive amplitudes.

We found that four operations are needed to go from the Drell-Yan-West frame [1] to the Breit frame [3]. Combining the four operations, we find the equivalent single operation that can go back and forth between two frames: $U = \exp[i(\alpha \mathcal{J}^+ + \beta \mathcal{K}^3)]$, where the coefficients α and β of the transverse rotation $\mathcal{J}^+$ and the boost $\mathcal{K}^3$ are functions of q^2. It turns out that the single operation is dynamical (not kinematical) because transverse rotation is involved in the single transformation. This means that the two reference-frames are not only kinematically different but also dynamically inequivalent even though both frames have $q^+ = 0$. Thus, some new dynamics can come in by moving from the Drell-Yan-West frame to the Breit frame. This in principle raises a flag on various form factor calculations performed in the Breit frame because the simple convolution formulas that worked out quite well in the Drell-Yan-West frame may not apply in the Breit frame. We thus investigate in detail both pseudoscalar and vector form factors in these two typical light-front frames. We find that the helicities in the Drell-Yan-West frame are not changed in the Breit frame and thus the form factors obtained in the two frames must be indeed identical[1]. We find that the

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[1] However, this doesn't mean that any model calculations of form factors would give the same

plus component of the Drell-Yan-West frame is proportional to the plus component of the Breit frame so that the same convolution formula can be used in both frames. This is a highly non-trivial result because such coincidence cannot be in general anticipated. We have also expanded the operation in a perturbative way. Closed exact single operation results $U = \exp[i(\alpha\mathcal{J}^+ + \beta\mathcal{K}^3)]$ were compared with the perturbative results.

In general, however, the light-front helicities are changed and the plus component of the current is mixed with other components under the change of reference frame. For example, the V-transformation was obtained as $V = \exp[i(-\alpha\mathcal{K}^- - \beta\mathcal{K}^3)]$ where $\mathcal{K}^-$ is the light-front transverse boost so that V commutes with the Hamiltonian. This reveals that the obtained V-transformation is dynamically equivalent even though it is kinematically different. In this case, however, the plus component of the current in Drell-Yan-West frame is not proportional to that in the V-frame. Thus, the same convolution formula obtained in the Drell-Yan-West frame cannot be used in the V-frame.

In the following section (Section II), we present the four operations needed to go from the Drell-Yan-West frame to the Breit frame and obtain the explicit relations for the matrix elements of the current in the two frames, confirming the usual unitary tranformation rule of the vector operator. It is also assured that the pseudoscalar form factor must be identical in the two frames because the plus component is preserved. In Section III, we apply the transformation to the helicities and show that the vector form factors must be also identical in the two frames because the helcities are not changed in the two frames. In Section IV, the V-transformation is presented to show that the usual convolution formula for the form factor calculation in the Drell-Yan-West frame cannot be used even though the transformation commutes with the hamiltonian involving dynamics. Conclusions and discussions follow in Section V. In the Appendix, the perturbative expansion of the transformation between the frames using the Campbell-Baker-Hausdorff relation is compared with the exact closed form of transformation.

II. TRANSFORMATION BETWEEN DRELL-YAN-WEST AND BREIT FRAMES

We begin by considering the absorption of a photon by a meson of mass M. The Drell-Yan-West reference frame is defined such that the initial four-momentum of the particle, p, and the four-momentum of the photon, q, are given as follows:

$$p = (p_0, 0, 0, |\mathbf{p}|) \tag{2.1}$$
$$q = (\frac{-q^2}{2p^+}, \sqrt{-q^2}, 0, \frac{q^2}{2p^+}).$$

We found the four transformations required to move from the Drell-Yan-West frame to the Breit frame. These are represented schematically in Fig. 1. The first transformation to

results in the two frames because the phenomenological models may not be covariant. For example, the angular condition for the vector meson form factors is violated if the model calculation is not fully covariant. Thus, our finding here provides an additional condition that the covariant model must satisfy besides the angular condition.
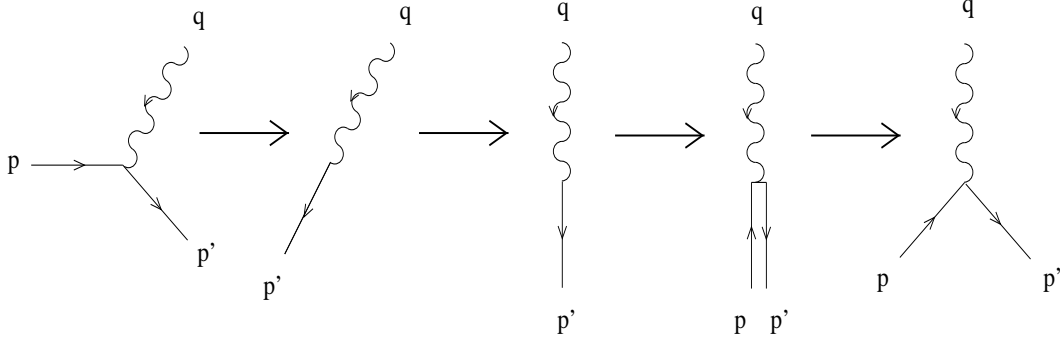
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DYW - Breit Transformations

FIG. 1. The four transformations going from the Drell-Yan-West frame to the Breit ($q^+ = 0$) frame.

the Breit frame is the boost in the z-direction which eliminates the three-momentum of the meson. The necessary boost parameter ω must satisfy $\sinh \omega = \frac{|\mathbf{p}|}{M}$, generating the Lorentz transformation

$$
\Lambda_1 = \begin{pmatrix}
\frac{E}{M} & 0 & 0 & -\frac{|\mathbf{p}|}{M} \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
-\frac{|\mathbf{p}|}{M} & 0 & 0 & \frac{E}{M}
\end{pmatrix}.
\tag{2.2}
$$

The momenta in the resulting frame are $p = (M, 0, 0, 0)$ and $q = (\frac{-q^2}{2M}, \sqrt{-q^2}, 0, \frac{q^2}{2M})$.

The second transformation is a rotation about the y-axis so that both the final momentum of the massive particle and the momentum of the photon lie along the x-axis. This requires that $\tan \theta = \frac{\sqrt{-q^2}}{2M}$. Throughout this paper we will refer to the quantity $\frac{\sqrt{-q^2}}{2M}$ as κ. Therefore,

$$
\Lambda_2 = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & \frac{1}{\sqrt{1+\kappa^2}} & 0 & \frac{-\kappa}{\sqrt{1+\kappa^2}} \\
0 & 0 & 1 & 0 \\
0 & \frac{\kappa}{\sqrt{1+\kappa^2}} & 0 & \frac{1}{\sqrt{1+\kappa^2}}
\end{pmatrix}.
\tag{2.3}
$$

The initial momentum is unchanged, but the photon momentum is now $q = (2M\kappa^2, 2M\kappa\sqrt{1+\kappa^2}, 0, 0)$ and the final momentum is $p' = (M + 2M\kappa^2, 2M\kappa\sqrt{1+\kappa^2}, 0, 0)$.

The next transformation is a boost by parameter ω_2 in the x-direction into a frame in which the incoming and outgoing momenta of the particle are equal and opposite, *i.e.*, $\mathbf{p} = -\mathbf{p}'$. Now $\sinh \omega_2 = \kappa$, so that

$$
\Lambda_3 = \begin{pmatrix}
\sqrt{1+\kappa^2} & -\kappa & 0 & 0 \\
-\kappa & \sqrt{1+\kappa^2} & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}.
\tag{2.4}
$$

4

Now the momenta are $p = (M\sqrt{1+\kappa^2}, -M\kappa, 0, 0)$, $q = (0, 2M\kappa, 0, 0)$, and $p' = (M\sqrt{1+\kappa^2}, M\kappa, 0, 0)$.

The final transformation is a boost opposite the initial one by the parameter $-\omega$ generating

$$\Lambda_4 = \begin{pmatrix} \frac{E}{M} & 0 & 0 & \frac{|\mathbf{p}|}{M} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \frac{|\mathbf{p}|}{M} & 0 & 0 & \frac{E}{M} \end{pmatrix}. \tag{2.5}$$

The resulting reference frame is known as the Breit frame, with new momenta: $p = (p^0\sqrt{1+\kappa^2}, -M\kappa, 0, |\mathbf{p}|\sqrt{1+\kappa^2})$ and $q = (0, 2M\kappa, 0, 0)$. If we define $Q = \sqrt{-q^2}$, the momenta in the light-front formalism $k = (k^+, k^-, \mathbf{k})$ become

$$p = (p_i^+ \sqrt{1+\kappa^2}, p_i^- \sqrt{1+\kappa^2}, -\frac{Q}{2}, 0) \tag{2.6}$$

$$q = (0, 0, Q, 0)$$

$$p' = (p_i^+ \sqrt{1+\kappa^2}, p_i^- \sqrt{1+\kappa^2}, \frac{Q}{2}, 0)$$

where p_i is the initial momentum of the massive particle in the Drell-Yan-West frame.

The product of these four unitary transformations is a single unitary transformation, U, which characterizes the relationship between the two frames. To fully understand the invariance of electromagnetic form factors under U, we must know how the matrix elements of the current transform. We explicitly verified that $U^\dagger I^\mu U = \Lambda^\mu_\nu I^\nu$, where Λ is the full Lorentz transformation matrix. Applying this matrix to an arbitrary current four-vector I^μ allows us to find the current-operator relations:

$$U^\dagger I^+ U = \sqrt{1+\kappa^2} I^+ \tag{2.7}$$

$$U^\dagger I^- U = \frac{1}{\sqrt{1+\kappa^2}} I^- + \frac{p_i^- \kappa}{M\sqrt{1+\kappa^2}} \left(\frac{p_i^- \kappa}{M} I^+ - 2I^1 \right)$$

$$U^\dagger I^1 U = I^1 - \frac{p_i^- \kappa}{M} I^+$$

$$U^\dagger I^2 U = I^2$$

Notice that the plus component of the current in Drell-Yan-West frame is proportional to the plus component of the current in the Breit frame. This correspondence is necessary if the convolution formalism is to be valid in both frames.

We can use these relations to find a closed form for the generator of the transformation. First, note that the operator must have the form $U = \exp i(\alpha \mathcal{J}^+ + \beta \mathcal{K}^3 + \gamma \mathcal{K}^-)$, where we have defined $\mathcal{J}^+ = \mathcal{J}^2 + \mathcal{K}^1$ and $\mathcal{K}^- = \mathcal{K}^1 - \mathcal{J}^2$ to be components of angular momentum and boost on the light-front, respectively. The coefficient γ must be zero according to arguments presented in the Appendix. In Fig. 2, the first (dotted) and second (dashed-dotted) order perturbative expansions for γ are plotted versus the parameter κ to demonstrate that the expansion converges to $\gamma = 0$. Next, by using the Campbell-Baker-Hausdorff relation with the appropriate Poincare algebra we find
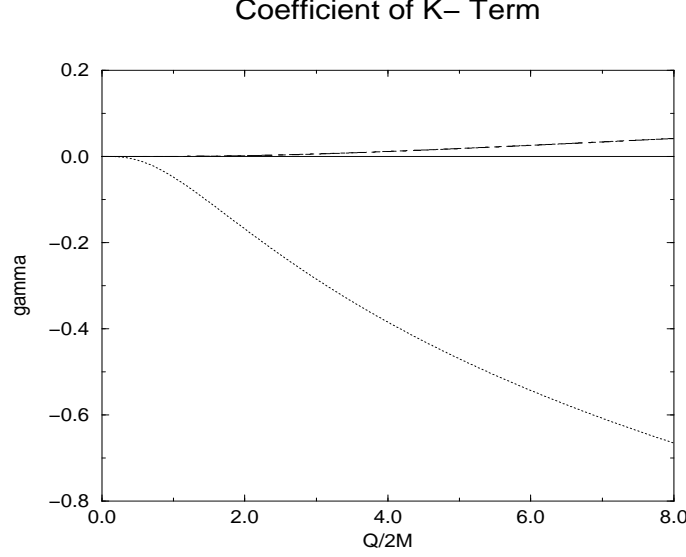
Coefficient of K– Term

FIG. 2. Comparison between the perturbative results of γ and the exact result $\gamma = 0$.

$$U^\dagger I^+ U = I^+ e^\beta \qquad (2.8)$$

$$U^\dagger I^1 U = I^1 + I^+ \frac{\alpha}{\beta}\left(e^\beta - 1\right).$$

Thus the full transformation from the Drell-Yan-West frame to the Breit frame can be represented by the unitary transformation

$$U = \exp i\left(\alpha \mathcal{J}^+ + \beta \mathcal{K}^3\right) \qquad (2.9)$$

where

$$\alpha = \frac{\kappa}{1 - \sqrt{1 + \kappa^2}} \frac{p_i^-}{M} \ln\sqrt{1 + \kappa^2} \qquad (2.10)$$

$$\beta = \ln\sqrt{1 + \kappa^2}.$$

This result was checked explicitly by both construction of the corresponding Lorentz transformation matrix and comparison with the perturbative expansion of $e^{i\omega\mathcal{K}^3} e^{-i\omega_2\mathcal{K}^1} e^{-i\theta\mathcal{J}^2} e^{-i\omega\mathcal{K}^3}$ as presented in the Appendix. Plots of the coefficients α and β are included for comparison with the perturbative expansion with $\omega = 0$ in Figs. 3 and 4. Solid lines correspond to exact solutions; dotted lines are first-order expansions; and dashed-dotted lines are up to third-order expansions. Note that the second-order terms are absent as shown in the Appendix if $\omega = 0$ (See Eq.(A.4)). The dashed-dotted line is indistinguishable from the solid line in Fig. 4.

To verify the invariance of the pseudoscalar form factor under this transformation, consider the relation $< p'|I^\mu|p > = (p + p')^\mu F$, where the matrix element and the momenta are defined in the Drell-Yan-West frame. In the Breit frame this becomes $< p'_D|U^\dagger I^\mu U|p_D > = \Lambda^\mu_\nu < p'_D|I^\nu|p_D > = \Lambda^\mu_\nu (p+p')^\nu_D F = (p+p')^\mu_B F$. Since the relation must hold in any reference frame, we see that the form factor F is invariant under Lorentz transformations. This can
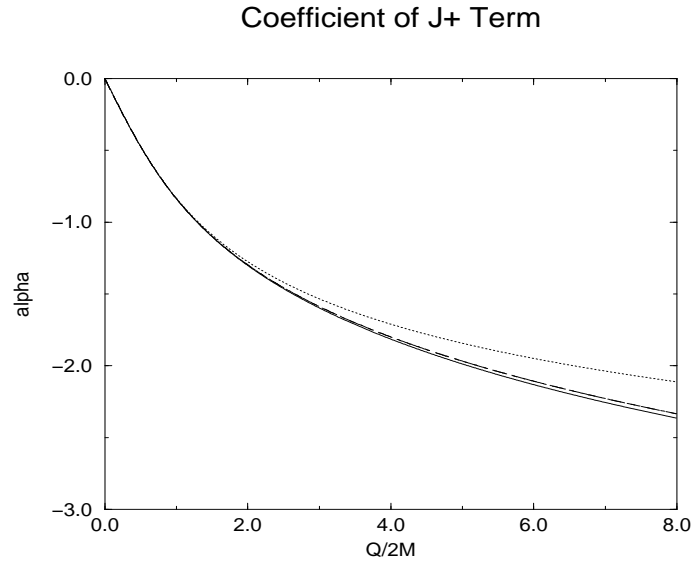
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Coefficient of J+ Term

FIG. 3. Comparison of α between the perturbative results and the exact result.
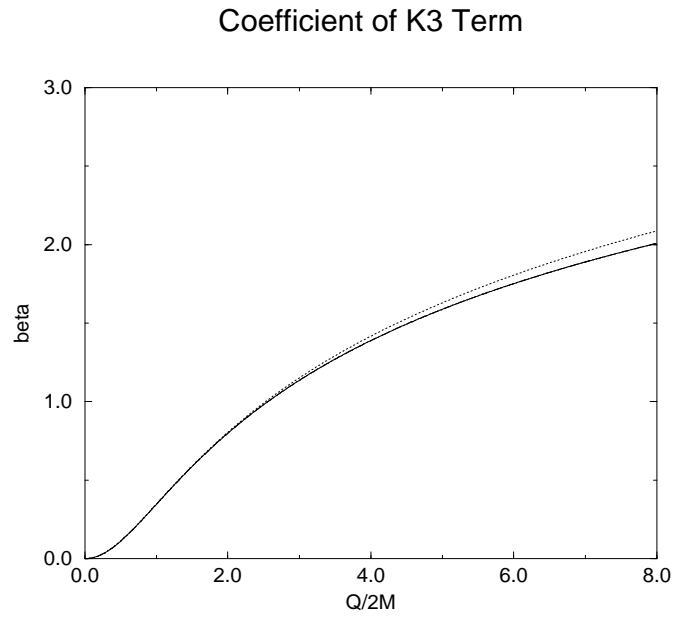
Coefficient of K3 Term

FIG. 4. Comparison of β between the perturbative results and the exact result.

be easily confirmed for the U transformation by using the current-operator relations given by Eq.(2.7).

More importantly, the Drell-Yan-West particle-number-conserving convolution formalism will be valid in the Breit frame. This is true since the plus component of any four-vector (*i.e.*, current) is unmixed with other components under this change of reference frame indicating $q^+ = 0 \to q^+ = 0$. Thus, the non-valence (pair-creation) diagrams remain excluded.

III. TRANSFORMATION OF HELICITIES AND THE EQUIVALENCE OF VECTOR FORM FACTORS

The invariance of vector or spin-one form factors is nontrivial due to the frame-dependence of light-front helicities. In this section we find that light-front helicity eigenvalues are unchanged under transformation from the Drell-Yan-West to the Breit reference frame.

The polarization vectors for light-front helicity eigenstates in the Drell-Yan-West frame are given by

$$\epsilon(0) = \frac{1}{M}(p_i^+, -p_i^-, 0, 0) \tag{3.1}$$

$$\epsilon(+1) = -\frac{1}{\sqrt{2}}(0, 0, 1, i)$$

$$\epsilon(-1) = \frac{1}{\sqrt{2}}(0, 0, 1, -i)$$

$$\epsilon'(0) = \frac{1}{M}(p_i^+, -p_i^- + \frac{Q^2}{p_i^+}, Q, 0)$$

$$\epsilon'(+1) = -\frac{1}{\sqrt{2}}(0, \frac{2Q}{p_i^+}, 1, i)$$

$$\epsilon'(-1) = \frac{1}{\sqrt{2}}(0, \frac{2Q}{p_i^+}, 1, -i)$$

while the corresponding vectors in the Breit frame are:

$$\epsilon(0) = \frac{1}{M}(p_i^+\sqrt{1+\kappa^2}, p_i^- \frac{\kappa^2-1}{\sqrt{1+\kappa^2}}, -\frac{Q}{2}, 0) \tag{3.2}$$

$$\epsilon(+1) = -\frac{1}{\sqrt{2}}(0, -\frac{Q}{p_i^+\sqrt{1+\kappa^2}}, 1, i)$$

$$\epsilon(-1) = \frac{1}{\sqrt{2}}(0, -\frac{Q}{p_i^+\sqrt{1+\kappa^2}}, 1, -i)$$

$$\epsilon'(0) = \frac{1}{M}(p_i^+\sqrt{1+\kappa^2}, p_i^- \frac{\kappa^2-1}{\sqrt{1+\kappa^2}}, \frac{Q}{2}, 0)$$

$$\epsilon'(+1) = -\frac{1}{\sqrt{2}}(0, \frac{Q}{p_i^+\sqrt{1+\kappa^2}}, 1, i)$$

$$\epsilon'(-1) = \frac{1}{\sqrt{2}}(0, \frac{Q}{p_i^+\sqrt{1+\kappa^2}}, 1, -i).$$

In each frame, the polarization vector representing a particular helicity eigenstate is determined by applying the three constraints

$$\epsilon(p, \lambda) \cdot p = 0 \tag{3.3}$$

$$\epsilon^*(p, \lambda) \cdot \epsilon(p, \lambda) = -\delta_{\lambda\lambda'}$$

$$\sum_\lambda \epsilon^\mu(p, \lambda)\epsilon^\nu(p, \lambda) = -g^{\mu\nu} + \frac{p^\nu p^\mu}{M^2}$$

and choosing $\epsilon(p, 0)$ so that its plus and perpendicular components are proportional to the plus and perpendicular components of the momentum p. In general, any polarization vector in one frame transforms to a superposition of polarization vectors in a second reference frame. Thus, light-front helicities are not Lorentz invariant. Applying the transformations described in the previous section to the six polarization vectors in the Drell-Yan-West frame, however, we obtain exactly the corresponding six vectors in the Breit frame. This implies that a state of helicity λ in the Drell-Yan-West frame transforms to a state of the same helicity λ in the Breit frame.

One consequence of this special relationship between the Drell-Yan-West and Breit reference frames is that the form of the angular condition, which is used to check the accuracy of model vector meson wavefunctions, is identical in both reference frames. The angular condition in the Drell-Yan-West frame is given by [4]

$$\Delta(Q^2) = (1 + 2\kappa^2)F_{++}^+ + F_{+-}^+ - \kappa\sqrt{8}F_{+0}^+ - F_{00}^+ = 0, \tag{3.4}$$

where $F_{\lambda\lambda'}^+ = < p', \lambda'|I^+|p, \lambda >$. To see how Eq.(3.4) holds also in the Breit frame, first note that $< p'_B, \lambda'_B|I^\mu|p_B, \lambda_B > = < p'_D, \lambda'_D|U^\dagger I^\mu U|p_D, \lambda_D >$. Thus, each matrix element transforms according to the current-operator relations (see Eq.(2.7)) presented in the last section. Since the helicity is invariant under this particular transformation, the matrix elements in the Breit frame are given by

$$< p'_B, \lambda'_B|I^\mu|p_B, \lambda_B > = \sum_{m,n} < p', \lambda'(n)|U^\dagger I^\mu U|p, \lambda(m) > c_m c_n \tag{3.5}$$

$$= < p'_D, \lambda'_D|U^\dagger I^\mu U|p_D, \lambda_D > = \Lambda_\nu^\mu < p'_D, \lambda'_D|I^\nu|p_D, \lambda_D > .$$

This shows that $F_{\lambda\lambda'}^+ \rightarrow \sqrt{1 + \kappa^2}F_{\lambda\lambda'}^+$ under the transformation from the Drell-Yan-West frame to the Breit frame. Therefore, Eq.(3.4) must also hold in the Breit frame. Similarly, one can show not only that $q^+ = 0 \rightarrow q^+ = 0$ but also that the relations between the vector form factors $\{F_1, F_2, F_3\}$ and $F_{\lambda\lambda'}^+$ remain the same in both frames. This is a remarkable feature of the two frames which results in the invariance of the convolution formalism.

Furthermore, the matrix elements are related to the vector form factors as follows

$$< p', \lambda'|I^\mu|p, \lambda > = \epsilon_\alpha^{*'}\epsilon_\beta[-g^{\alpha\beta}(p + p')^\mu F_1 + (g^{\mu\beta}q^\alpha - g^{\mu\alpha}q^\beta)F_2 + \frac{q^\alpha q^\beta(p + p')^\mu F_3}{2M^2}]. \tag{3.6}$$

Under Lorentz transformations all scalar products on the right remain invariant, so that each term on the right-hand side transforms as the μ component of a momentum or polarization four-vector times a form factor. This means that in the Briet frame the matrix elements can be written as

$$< p'_B, \lambda'_B|I^\mu|p_B, \lambda_B > = \sum_i \rho_{Bi}^\mu F_i^B, \tag{3.7}$$

where $\rho^{\mu}_{Bi}(i = 1, 2, 3)$ is the coefficient of F_i (see Eq.(3.6)) in the Breit frame. According to Eq.(3.5), the left-hand side of Eq.(3.7) is also equal to

$$\Lambda^{\mu}_{\nu} < p'_D, \lambda'_D|I^{\nu}|p_D, \lambda_D > \ = \Lambda^{\mu}_{\nu} \sum_i \rho^{\mu}_{Di} F^D_i \tag{3.8}$$

$$= \sum_i \rho^{\mu}_{Bi} F^D_i.$$

Therefore, $\sum_i \rho^{\mu}_{Bi} F^B_i = \sum_i \rho^{\mu}_{Bi} F^D_i$. This can only be true if all three form factors F_1, F_2, F_3 are identical in both frames.

IV. THE V-TRANSFORMATION

It is well-known that helicities in the traditional equal-time quantization are frame dependent. We expect a similar phenomenon on the light-front. In order to demonstrate that light-front helicities are not frame-invariant in general, consider the following example. Define a transformation V which is identical to the previous transformation, U, except that each boost is reversed; i.e., $\omega \to -\omega$ and $\omega 2 \to -\omega 2$. Under this transformation we find the current-operator relations

$$V^{\dagger}I^-V = \sqrt{1 + \kappa^2}I^- \tag{4.1}$$

$$V^{\dagger}I^+V = \frac{1}{\sqrt{1 + \kappa^2}}I^+ + \frac{p^-_i \kappa}{M\sqrt{1 + \kappa^2}} \left(\frac{p^-_i \kappa}{M}I^- + 2I^1 \right)$$

$$V^{\dagger}I^1V = I^1 + \frac{p^-_i \kappa}{M}I^-$$

$$V^{\dagger}I^2V = I^2.$$

Note that the plus component of the current is not proportional to the plus component of the current in the Drell-Yan-West frame. Now applying the same constraints (3.3) as before, we obtain the polarization vectors for the initial meson:

$$\epsilon(0) = \frac{1}{M}(k^+, \frac{k_{\perp}{}^2 - M^2}{k^+}, k_{\perp}, 0) \tag{4.2}$$

$$\epsilon(+1) = -\frac{1}{\sqrt{2}}(0, \frac{2k_{\perp}}{k^+}, 1, i)$$

$$\epsilon(-1) = \frac{1}{\sqrt{2}}(0, \frac{2k_{\perp}}{k^+}, 1, -i),$$

where the parameters $k^+, k_{\perp}$ are components of the fermion's light-front momentum in the new V-frame which is given by

$$k = (\frac{p^-}{M^2\sqrt{1 + \kappa^2}} \left[(p^+)^2 + \kappa^2 (p^-)^2 \right], p^-\sqrt{1 + \kappa^2}, \frac{\kappa(p^-)^2}{M}, 0). \tag{4.3}$$

Similarly, the components of the photon's light-front momentum k_{γ} in the V-frame are

$$k_{\gamma} = (\frac{1}{p^+M^2\sqrt{1 + \kappa^2}} \left[M^2 + \kappa^2 (p^-)^2 \right], \frac{Q^2}{p^+}\sqrt{1 + \kappa^2}, \frac{Q}{p^+}(2x^2p^- + p^+), 0). \tag{4.4}$$

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The transformation V can be represented as the unitary operator $V = \exp i[-\alpha \mathcal{K}^- - \beta \mathcal{K}^3]$, where α and β were presented in section II.

Transforming the Drell-Yan-West polarization vectors according to V yields a set of polarization vectors in this new V-frame. None of these polarization vectors, however, represents a helicity eigenstate in this frame. We obtain, for example, that

$$\epsilon(0)^+ = \frac{p^-}{M^3\sqrt{1+\kappa^2}}\left[(p^+)^2 - \kappa^2(p^-)^2\right]. \tag{4.5}$$

The correct plus component of the zero-helicity polarization vector is $\frac{k^+}{M}$ from Eq.(4.2) or

$$\epsilon(0)^+ = \frac{p^-}{M^3\sqrt{1+\kappa^2}}\left[(p^+)^2 + \kappa^2(p^-)^2\right], \tag{4.6}$$

according to Eq.(4.3). The subtle sign difference is a consequence of changing the sign of each boost in the transformation. For a stronger example, consider the transverse polarization vectors. We obtain by transforming the Drell-Yan-West vectors

$$\epsilon(+1)^+ = -\frac{1}{\sqrt{2}}\left(\frac{2\kappa p^-}{M\sqrt{1+\kappa^2}}\right) \tag{4.7}$$

$$\epsilon(-1)^+ = \frac{1}{\sqrt{2}}\left(\frac{2\kappa p^-}{M\sqrt{1+\kappa^2}}\right).$$

These terms must be zero, as presented in (4.2), to satisfy the necessary constraints in the V-frame. Thus, light-front helicity is not invariant under this V-transformation. This implies that the convolution formulation used in the Drell-Yan-West frame cannot be used in the V-frame. As a result, we expect a different form for the angular condition in this frame.

V. CONCLUSION

We applied the four operations in the calculation of the pseudoscalar form factor and found that the form factor can be identically obtained in the Drell-Yan-West and Breit reference frames using the simple convolution formalism as long as the plus component of the current is used. We also applied the four operations in the calculation of spin-1 (vector) form factors, $F_1, F_2, F_3(G_E, G_M, G_Q)$. We found that the light-front helicities become identical in the two frames even though the light-front helicities are in general frame dependent. Thus, all three form factors obtained by the particle-number-conserving convolution formalism must be identical in the two frames confirming the correctness of previous applications to the vector meson form factors in the Breit frame. We also find that the angular condition is identical in the two frames. This is a remarkable result because it works only in very limited special frames. Thus, the Drell-Yan-West and Breit frames can be regarded as such special frames. However, such coincidence does not generally hold for other reference frames. One needs thus to investigate the frame-independence of form factors with care before relying on a particular reference frame. In summary, the two typical frames (Drell-Yan-West and Breit) are special in the light-front computation since the plus components of the current in the two

frames are proportional to each other and the light-front helicities are equivalent in these two frames. Thus, the angular condition is also identical and the same convolution formula obtained in the Drell-Yan-West frame can equivalently be used in the Breit frame. However, in other frames, caution is needed in using the Drell-Yan-West convolution formalism.

ACKNOWLEDGMENTS

APPENDIX A: COMPARISON WITH PERTURBATIVE EXPANSION USING CBH RELATIONS

According to the Campbell-Baker-Hausdorff Theorem, if $e^A e^B = e^C$ then C can be expressed as an expansion in terms of commutators. The first few terms are

$$C = A + B + \frac{1}{2}[A, B] + \frac{1}{12}([A, [A, B]] - [[A, B], A]) + \cdots. \tag{A1}$$

Applying this theorem three times allows us to combine the unitary transformation $e^{i\omega\mathcal{K}^3} e^{-i\omega_2 \mathcal{K}^1} e^{-i\theta\mathcal{J}^2} e^{-i\omega\mathcal{K}^3}$ into a single exponential, providing us with a perturbative expansion for the generator of the U-transformation.

$$C = \frac{1}{2}\mathcal{J}^+ (a + b)(1 - \frac{1}{2}\omega + \frac{1}{12}\omega^2 + \frac{1}{12}\omega c) + \frac{1}{2}\mathcal{K}^- (a - b)(1 - \frac{1}{2}\omega + \frac{1}{12}\omega^2 + \frac{1}{12}\omega c) \tag{A2}$$
$$+ \mathcal{K}^3 (c - \omega - \frac{1}{12}\omega(a^2 - b^2)) + \cdots.$$

Where $a, b,$ and c are given up to fourth order as

$$a = -\omega_2 + \frac{1}{2}\omega\theta + \frac{1}{12}\omega_2(-\omega^2 + \theta^2) + \frac{1}{12}(\omega_2{}^2\omega\theta) \tag{A3}$$
$$b = -\theta + \frac{1}{2}\omega\omega_2 - \frac{1}{12}\theta(\omega^2 + \omega_2{}^2)$$
$$c = \omega + \frac{1}{2}\theta\omega_2 + \frac{1}{12}\omega(-\theta^2 + \omega_2{}^2).$$

Suppose that $\omega = 0$. Then, keeping up to third order, we have

$$C = \frac{1}{2}\mathcal{J}^+ (-\omega_2 - \theta + \frac{1}{12}\theta^2\omega_2 - \frac{1}{12}\omega_2{}^2\theta) + \frac{1}{2}\mathcal{K}^- (-\omega_2 + \theta + \frac{1}{12}\theta^2\omega_2 + \frac{1}{12}\omega_2{}^2\theta) + \frac{1}{2}\mathcal{K}^3\omega_2\theta. \tag{A4}$$

Using the facts that $\tan\theta = \kappa$ and $\sinh\omega_2 = \kappa$, we can examine the coefficient of each operator versus the parameter κ. In Figs.2-4, each coefficient corresponding to $\alpha, \beta,$ and γ in Eq.(A.4) are compared with the closed form presented earlier (See Eq.(2.10)). For small values of κ the closed form and the expansion agree well, and their values diverge slowly with increasing κ as expected. The coefficient of the $\mathcal{K}^-$ operator approaches zero for a given κ as the order of approximation increases. One can indeed verify $\gamma = 0$ because we have already shown that the plus component of current in the Drell-Yan-West frame must be proportional to the plus component of current in the Breit frame (See Eq.(2.7)). Since the commutator $[I^+, \mathcal{K}^-] = 2iI^1$, including a $\mathcal{K}^-$ term in the transformation would contradict Eq.(2.7). Thus, the coefficient γ must be zero.

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